Autocorrelation image processing for shear analysis in weak lensing

Paul Stankus, ORNL

BNL Astro Group Meeting, 3 November 2015

Physics and shear

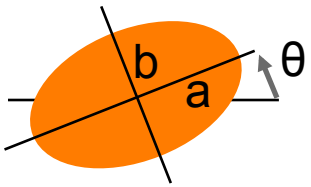
Cause:

$$\begin{bmatrix} 1+e1 & e2 \\ e2 & 1-e1 \end{bmatrix} \begin{pmatrix} x_{Unlens} \\ y_{Unlens} \end{pmatrix} = \begin{pmatrix} x_{Observ} \\ y_{Observ} \end{pmatrix}$$

(e1,e2) ←→ derivatives of gravitational lensing potential

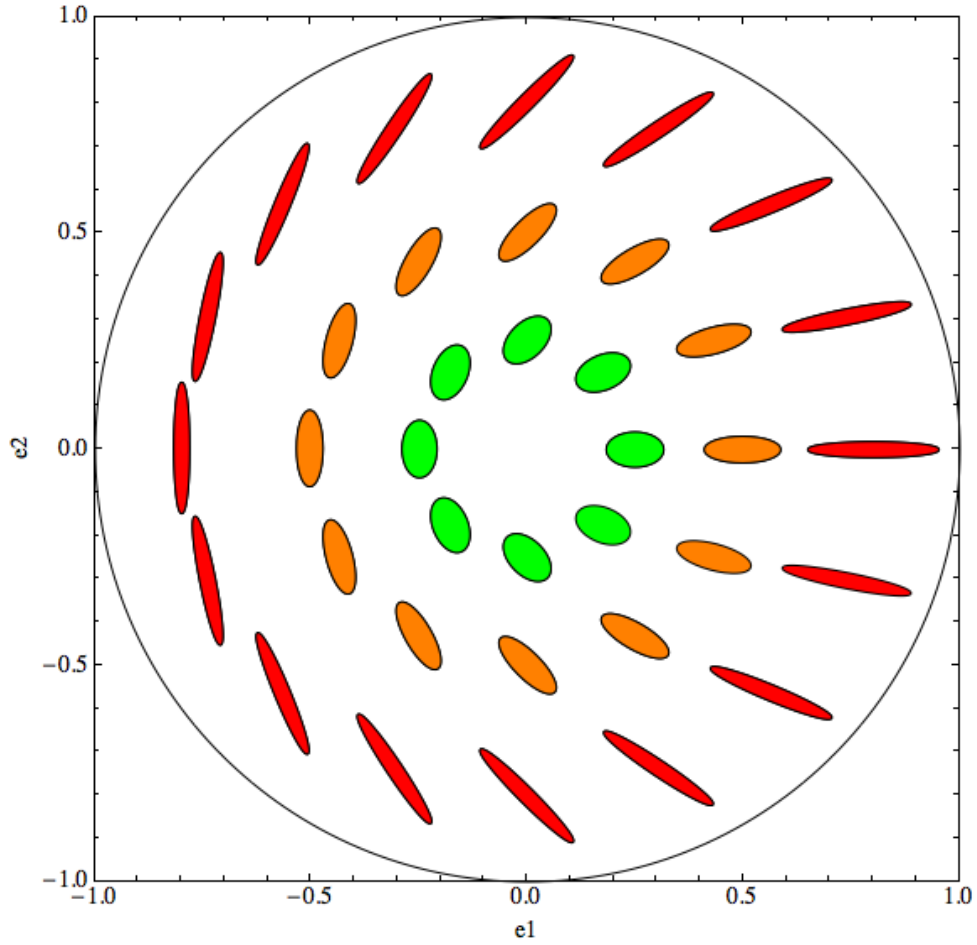
Effect:

$$\begin{Bmatrix} e1 \\ e2 \end{Bmatrix} = \begin{Bmatrix} e\,Cos\,(2\,\theta) \\ e\,Sin\,(2\,\theta) \end{Bmatrix}$$

$$\begin{bmatrix} 1+e1 & e2 \\ e2 & 1-e1 \end{bmatrix}$$  = 

$$e = \frac{a-b}{a+b}$$

Life in (e1,e2) space



e = 0.25, 0.5, 0.8

Measure (e1,e2) of galaxy images

→

Estimators of induced gravitational shear

Wish list?

Can we find an image processing technique that will let us

- Push every image toward a known fit-able shape, while still *preserving* eccentricity information

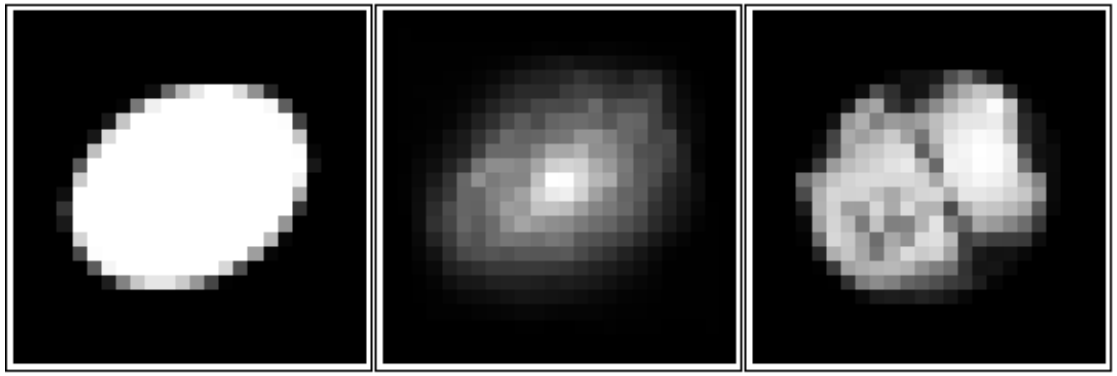
- Take advantage of the properties of pixel by pixel noise (whiter shade of pale)

Convolutions/Correlations

Two interesting, suggestive mathematical facts:

- Repeated auto-correlation/convolution converges toward a Gaussian, while preserving variance ratios
- Auto-correlation of white noise is a predictable $\delta()$ function

Image processing, first order

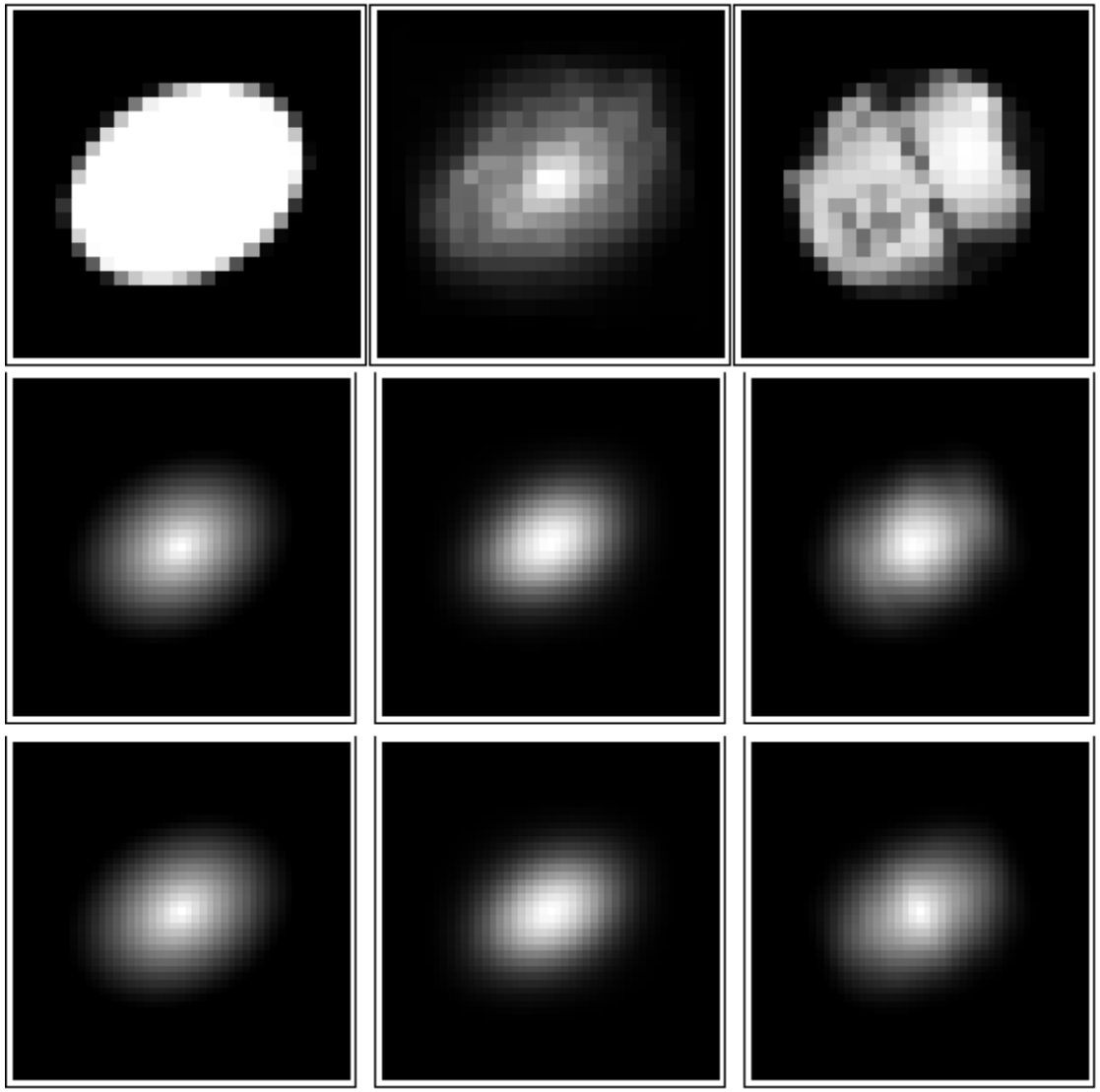


Original

Auto - convolution

Auto - correlation

Image processing, first order

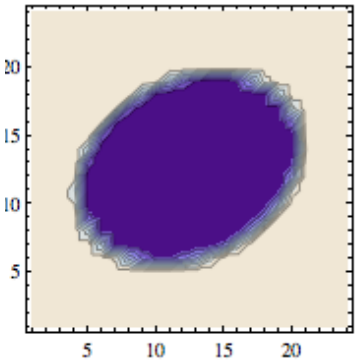


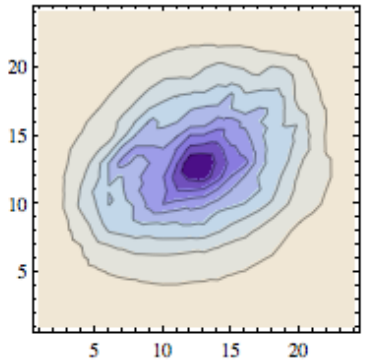
Original

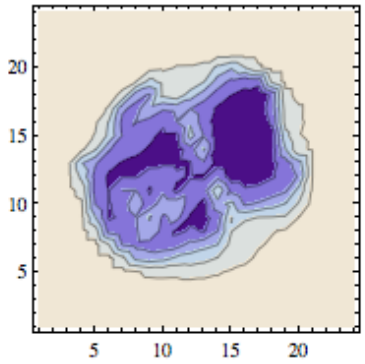
Auto - convolution

Auto - correlation

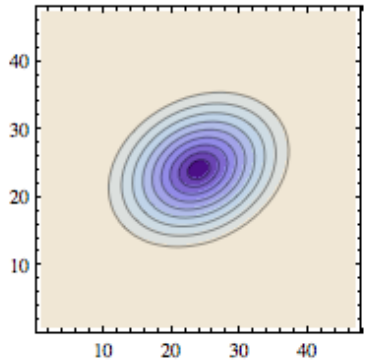
Successive autocorrelation

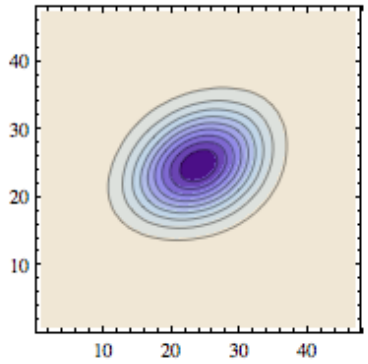


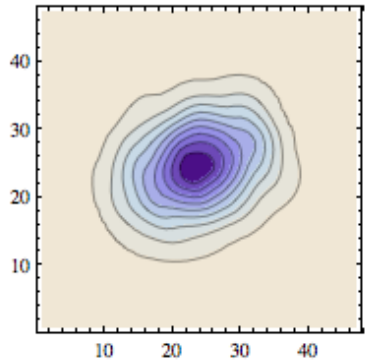




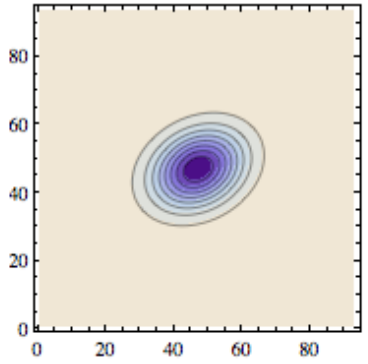
Original

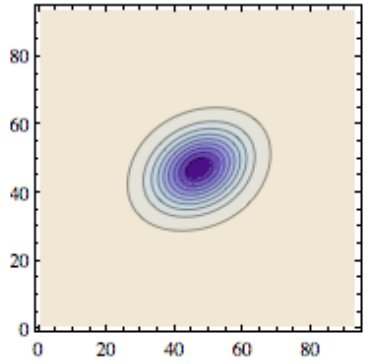


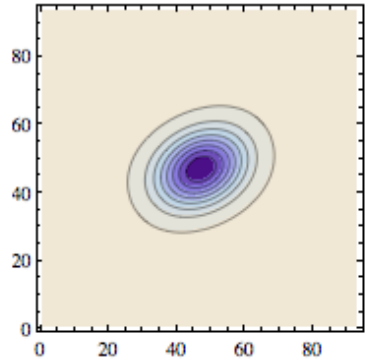




First auto - correlation

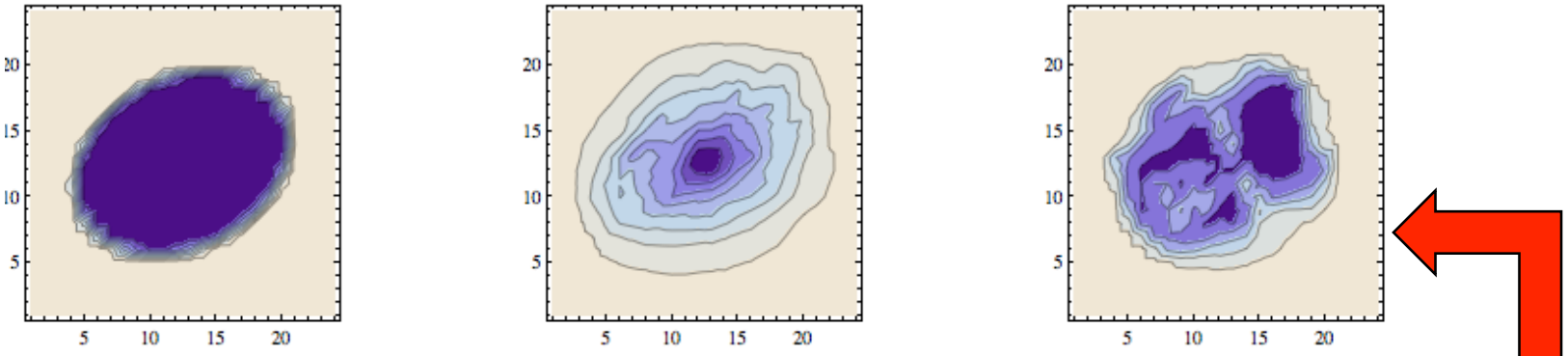






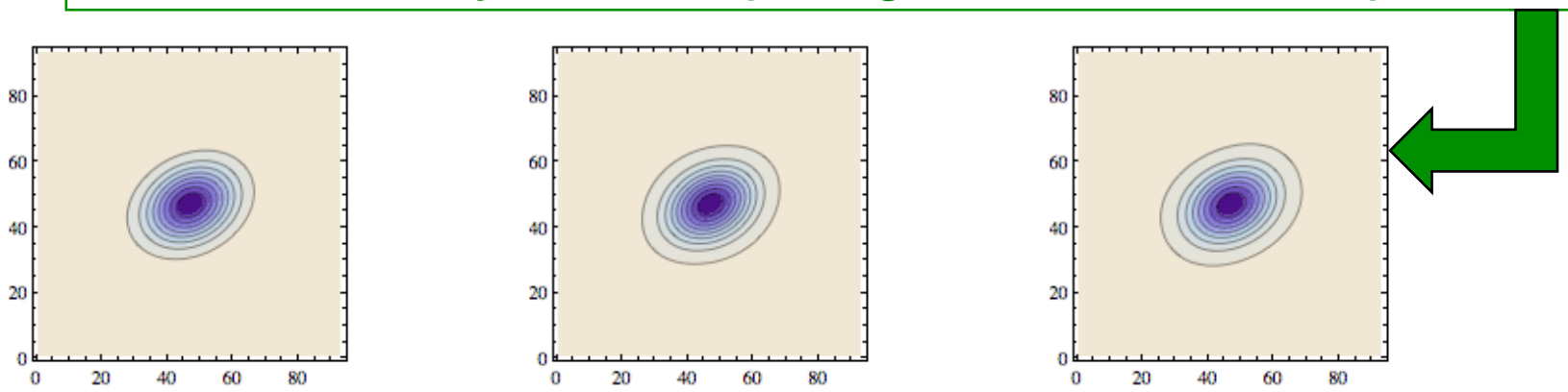
Second auto - correlation

Main Message:

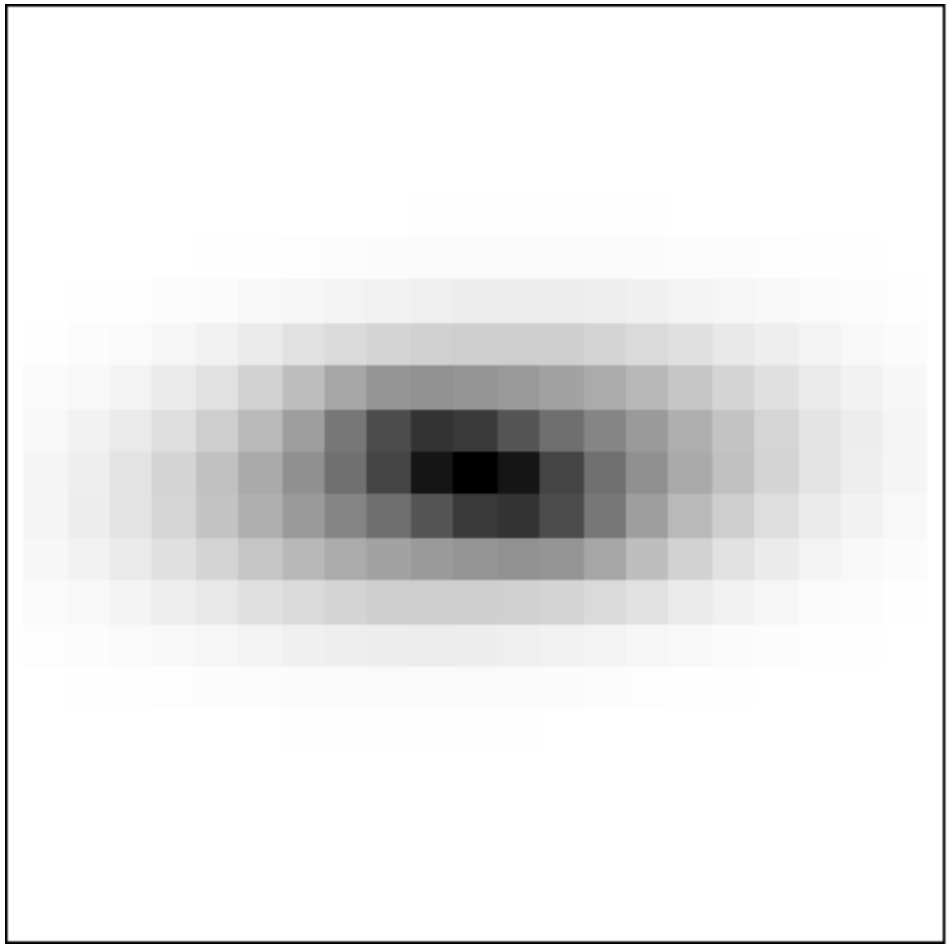


These are hard to fit, requiring a large “model space”

These are easy to fit, requiring a small “model space”



Example: Poor Man's Barred Spiral

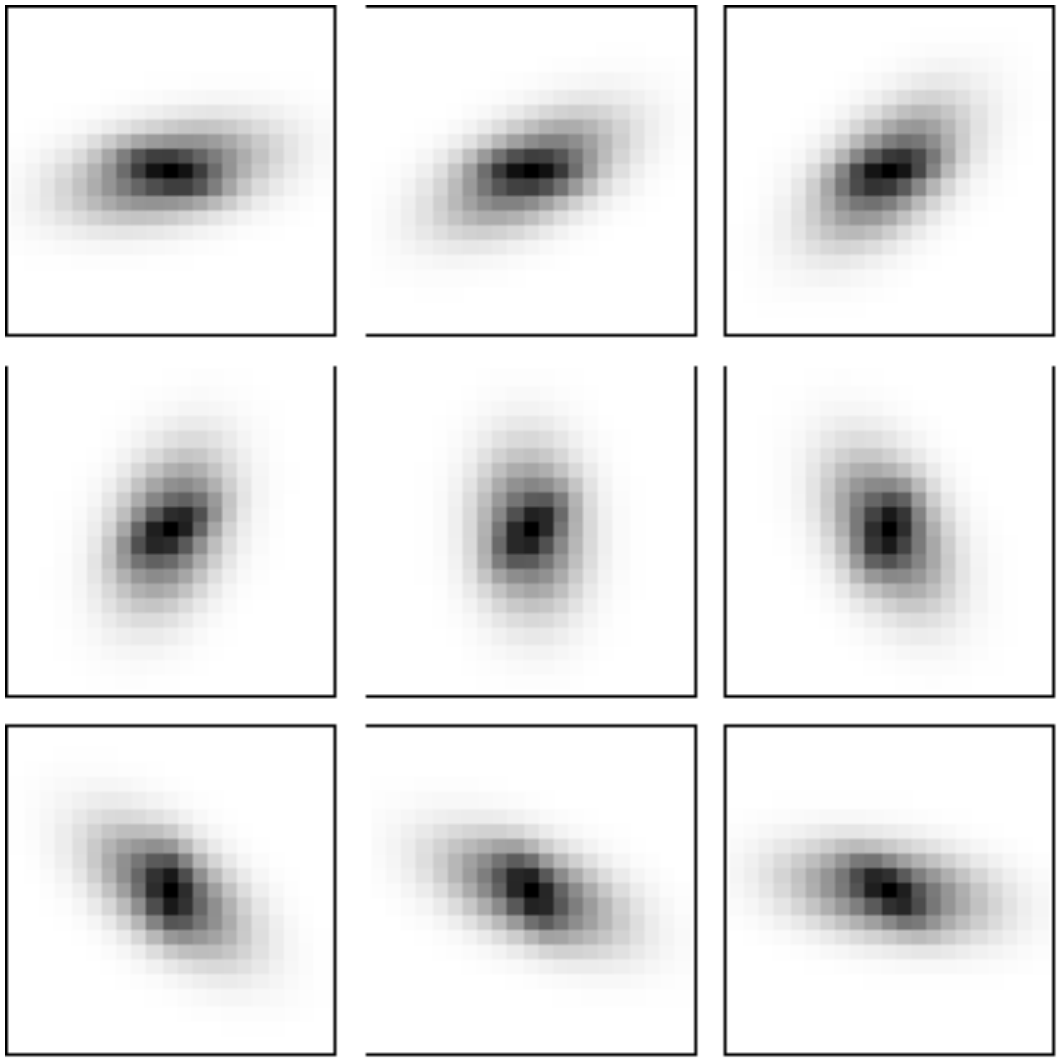


A galaxy-like image made from the sum of two Gaussians

Non-elliptical objects have no single, unambiguous intrinsic ellipticity

But all we require is a shear estimator

Shear Estimators



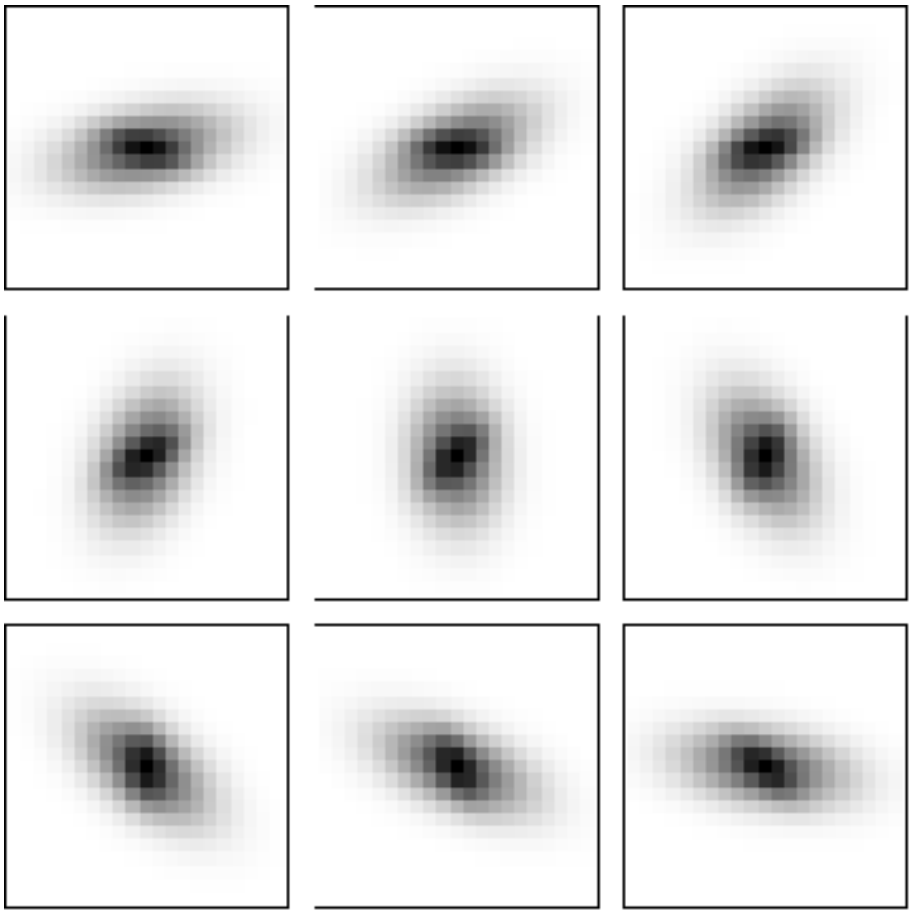
Take any image, and rotate it in many steps from 0-π

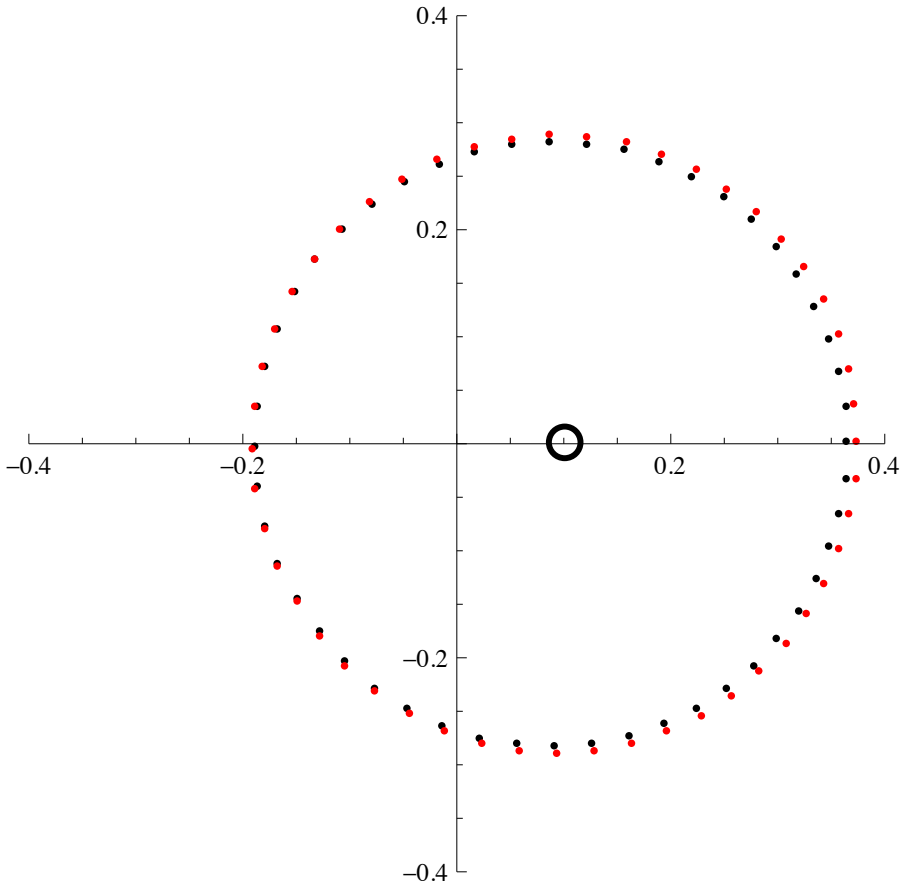
Shear each rotated image by the same given amount

A good shear estimator for each image, returns the induced shear when averaged over all rotations

Shear recovery

Each rotated image was sheared by {e1,e2}={0.1,0}, and the circle-averaged shear estimator recovers that





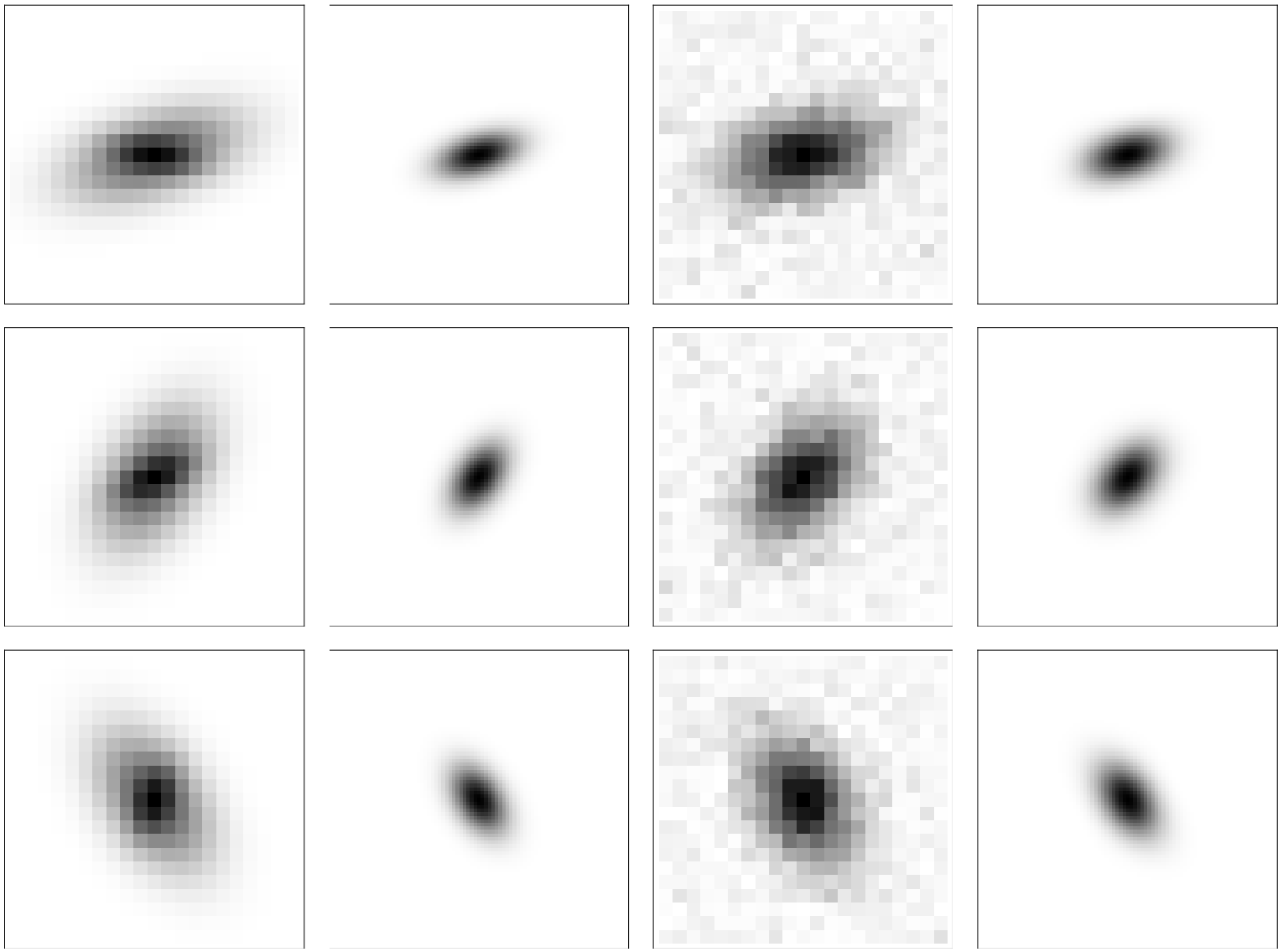
The challenge: noise and PSF

Two true facts:

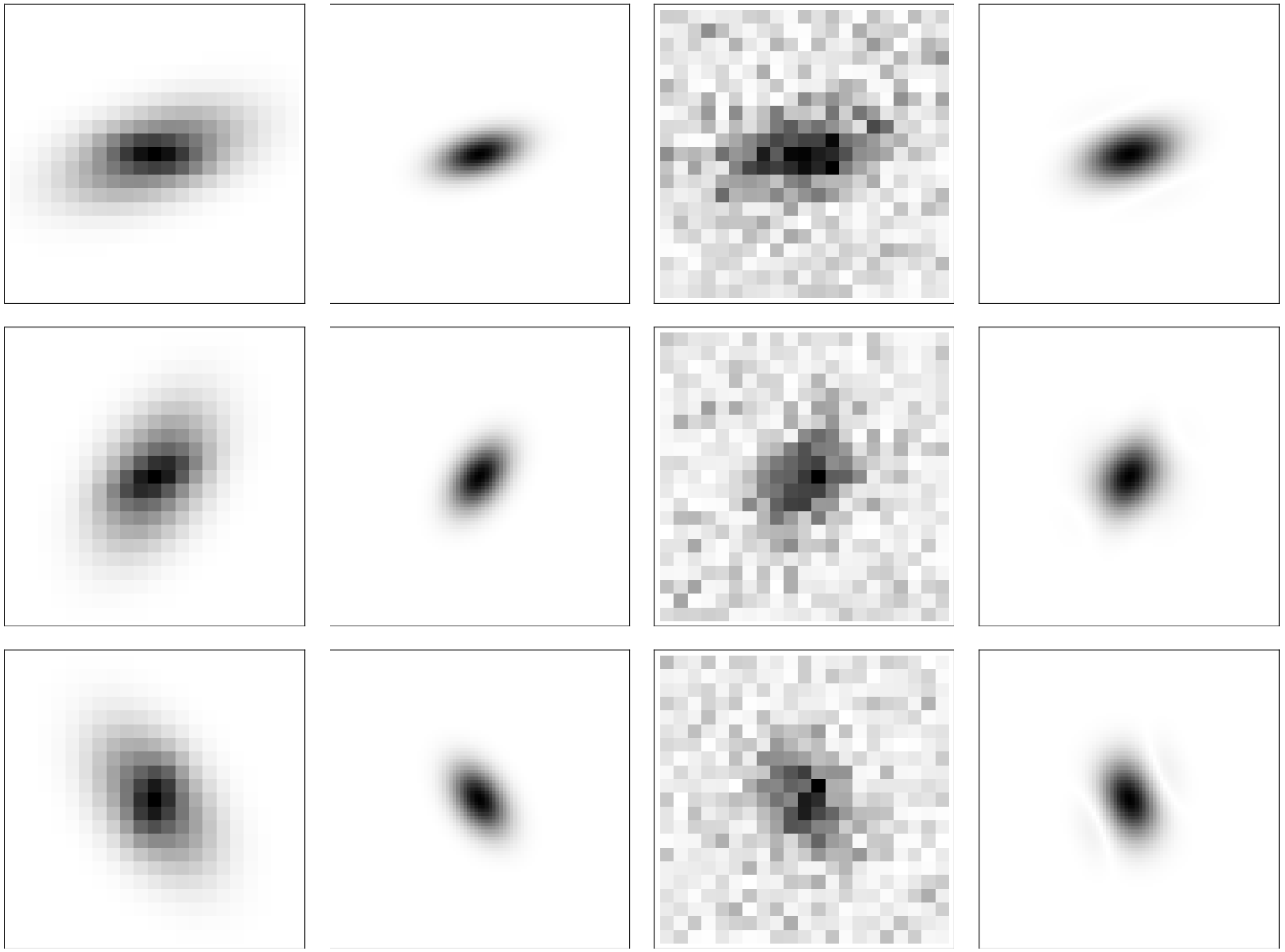
(1) Any auto-correlate of an image has the same shear transformation properties as images do

(2) If an observed image is the convolution of an original and a PSF, then its autocorrelate is the convolution of the original image autocorrelate and the PSF autocorrelate

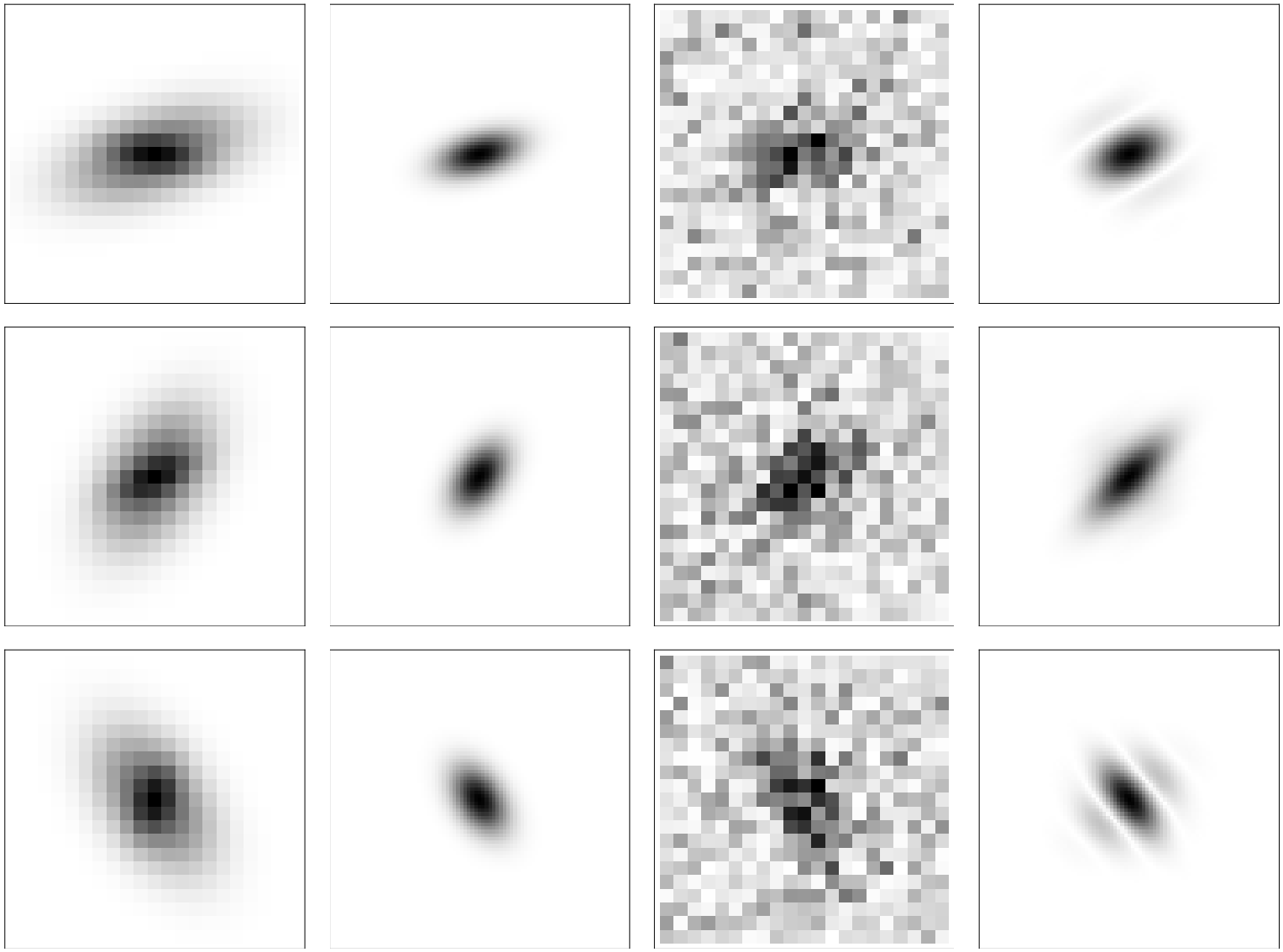

Original, rotated & sheared
Second autocorrelate
Add PSF and noise
Second autocorrelate
S/N = 10000
PSF = 1 pixel


S/N = 100 PSF = 1 pixel

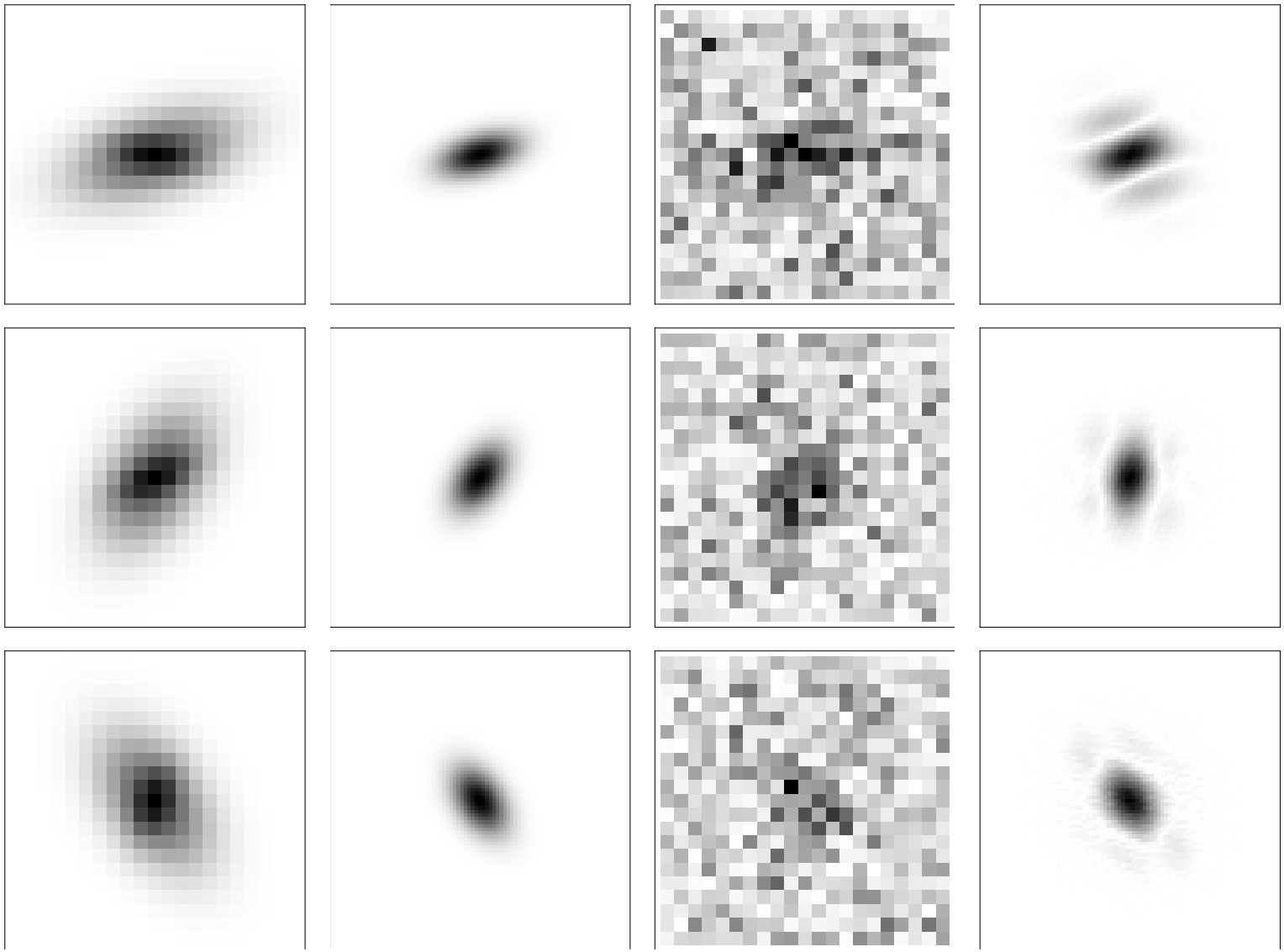
S/N = 30 PSF = 1 pixel



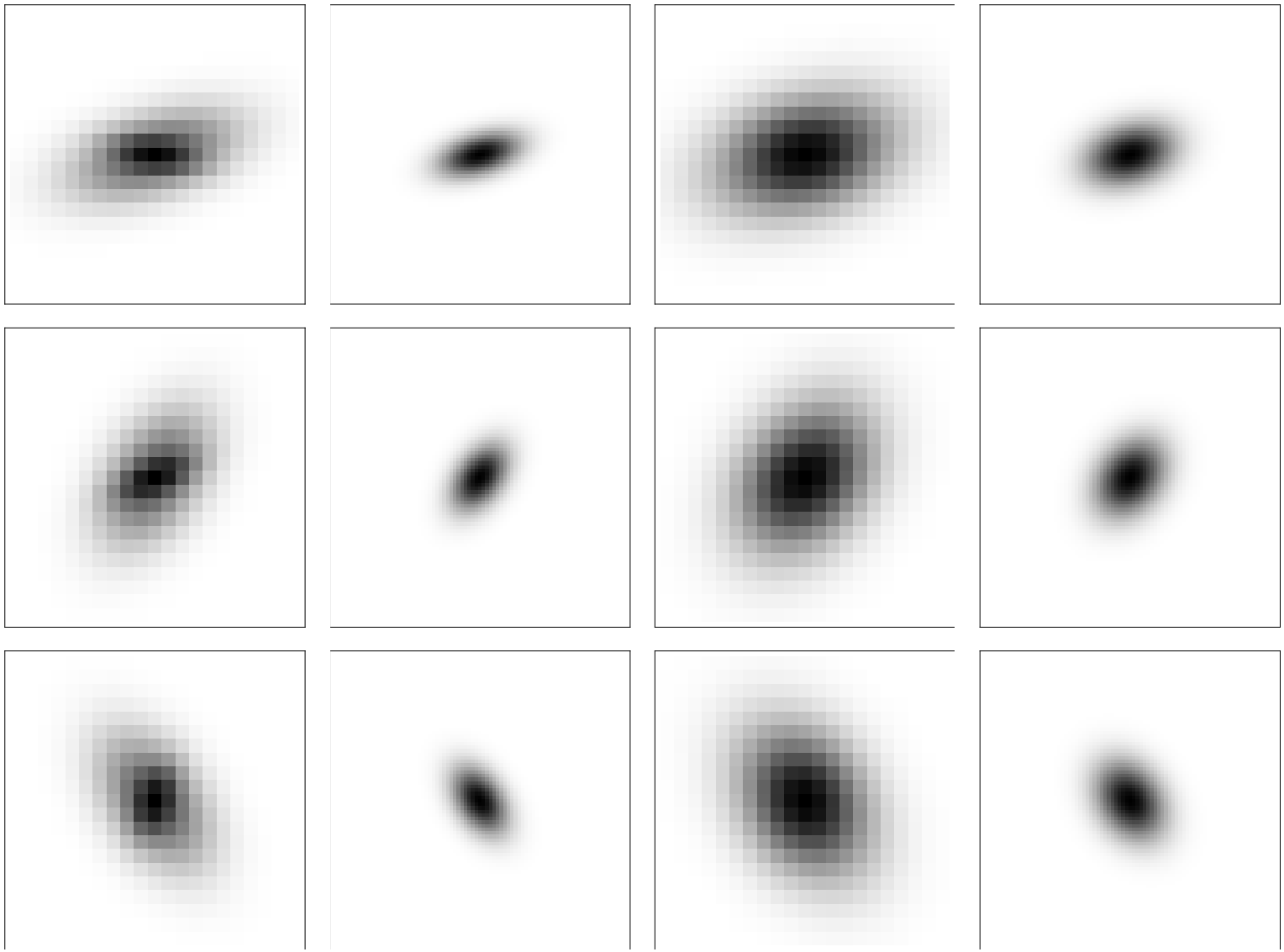
S/N = 20 PSF = 1 pixel



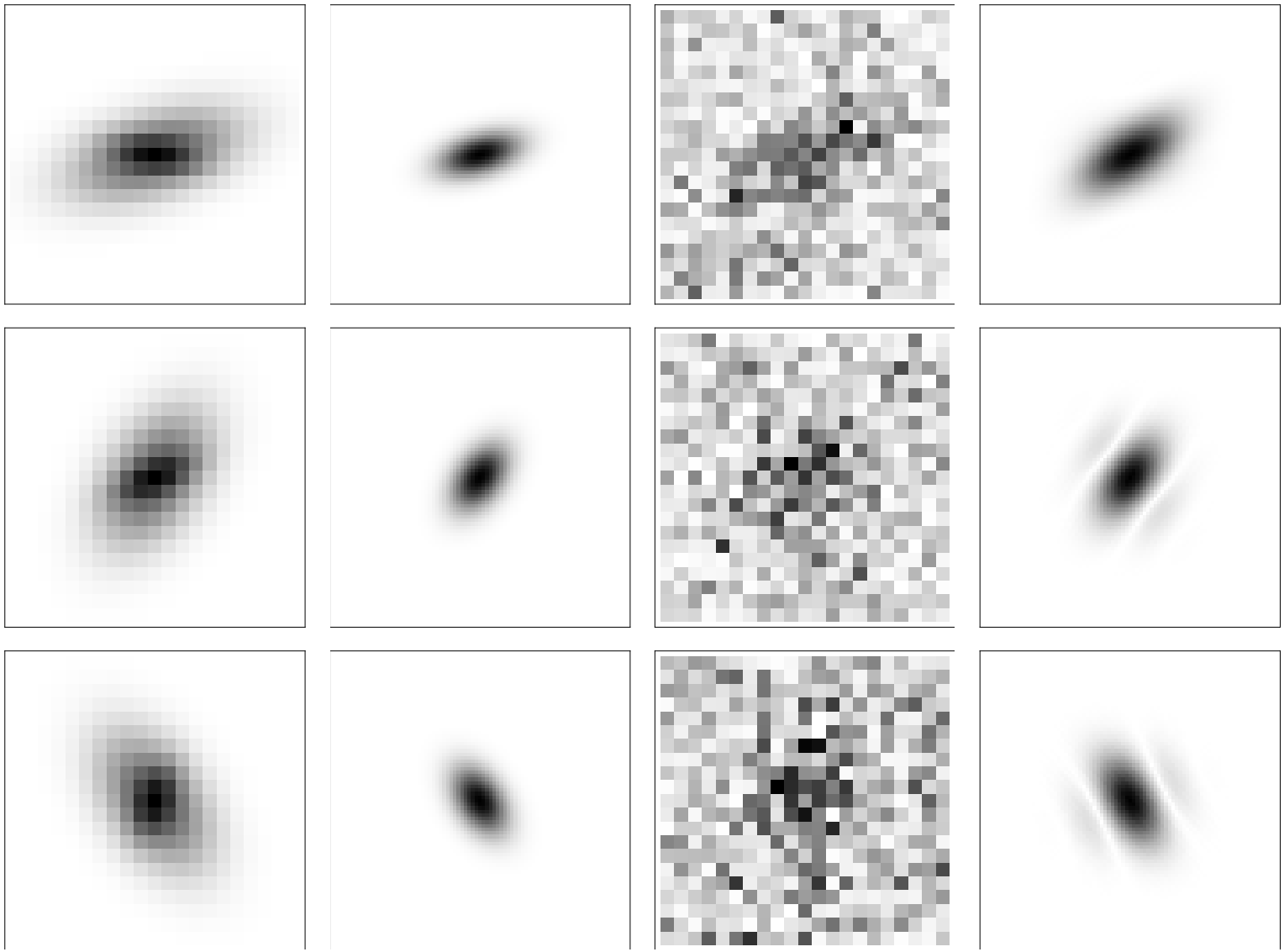
S/N = 15 PSF = 1 pixel



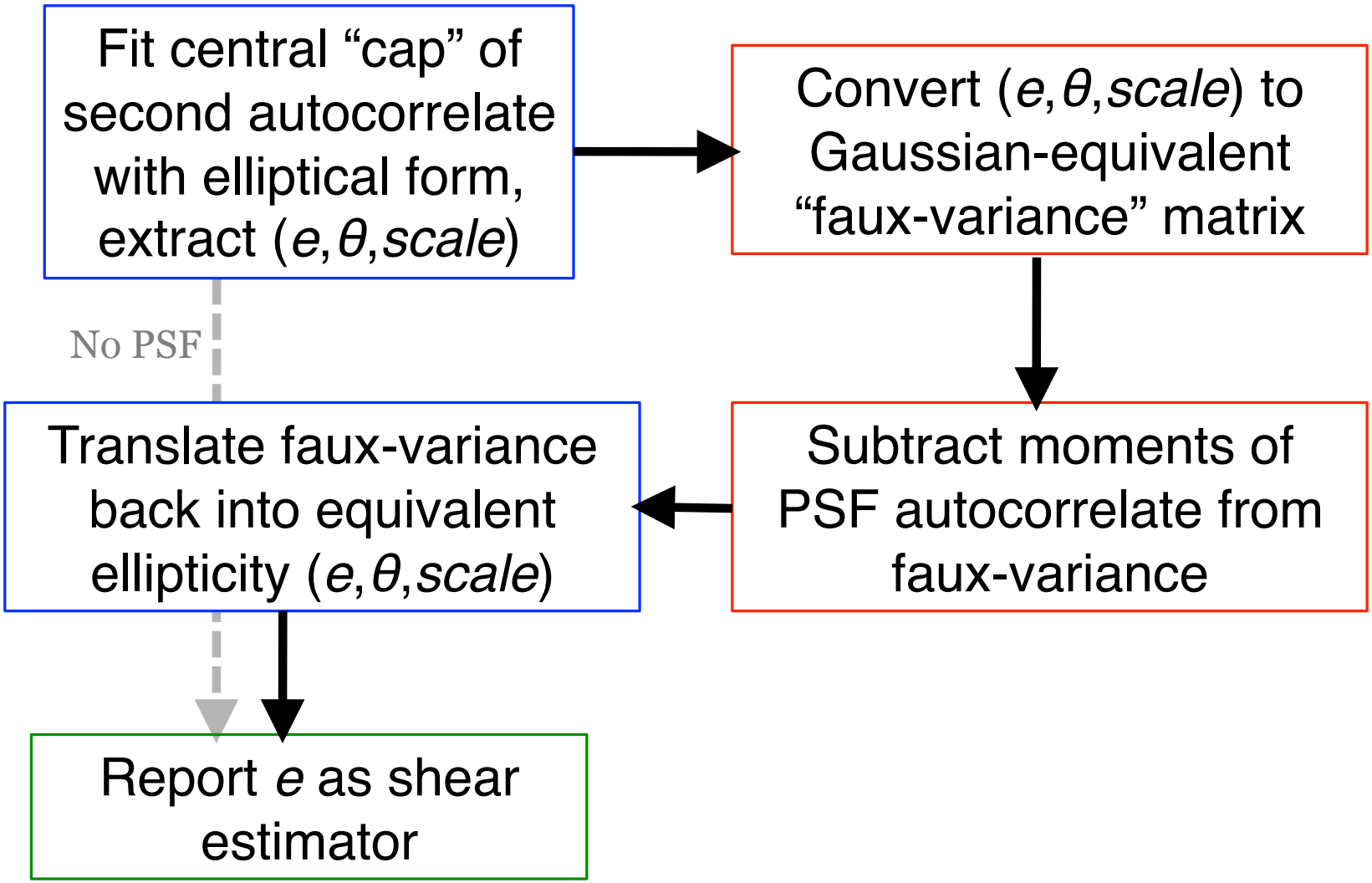
S/N = 10000 PSF = 2 pixel

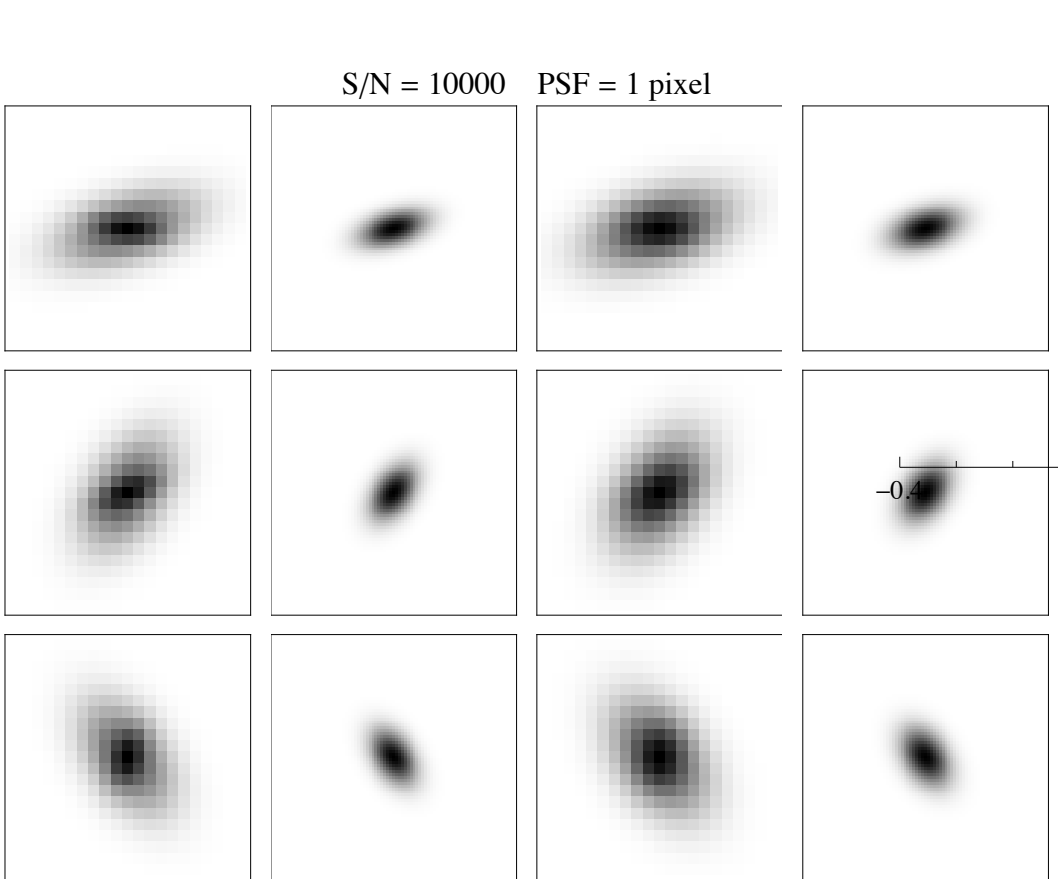


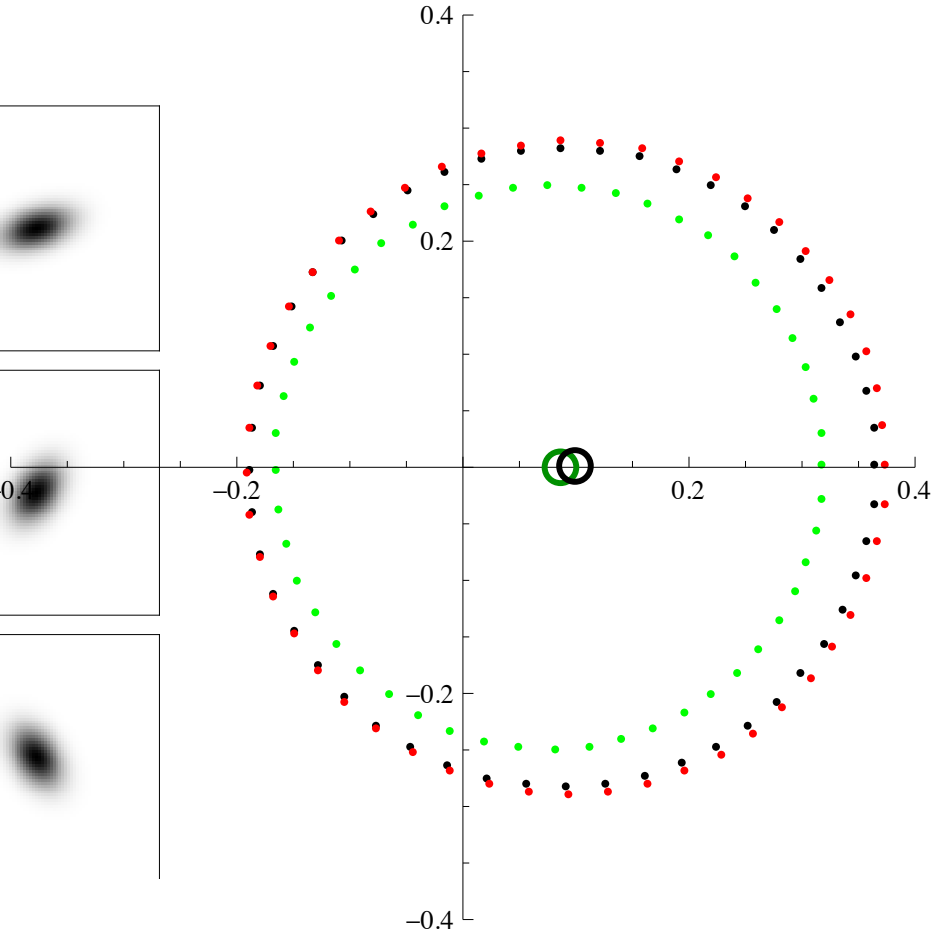
S/N = 15 PS F= 2 pixel



“Quickie” PSF correction



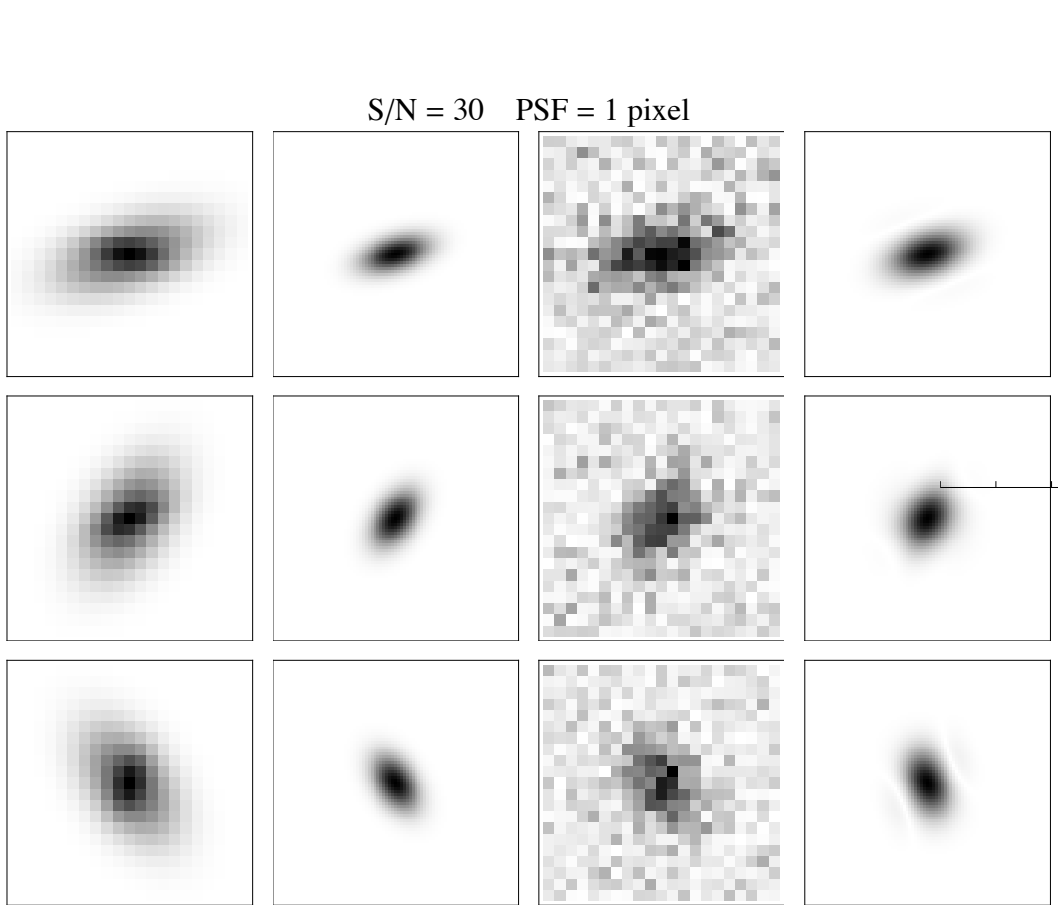


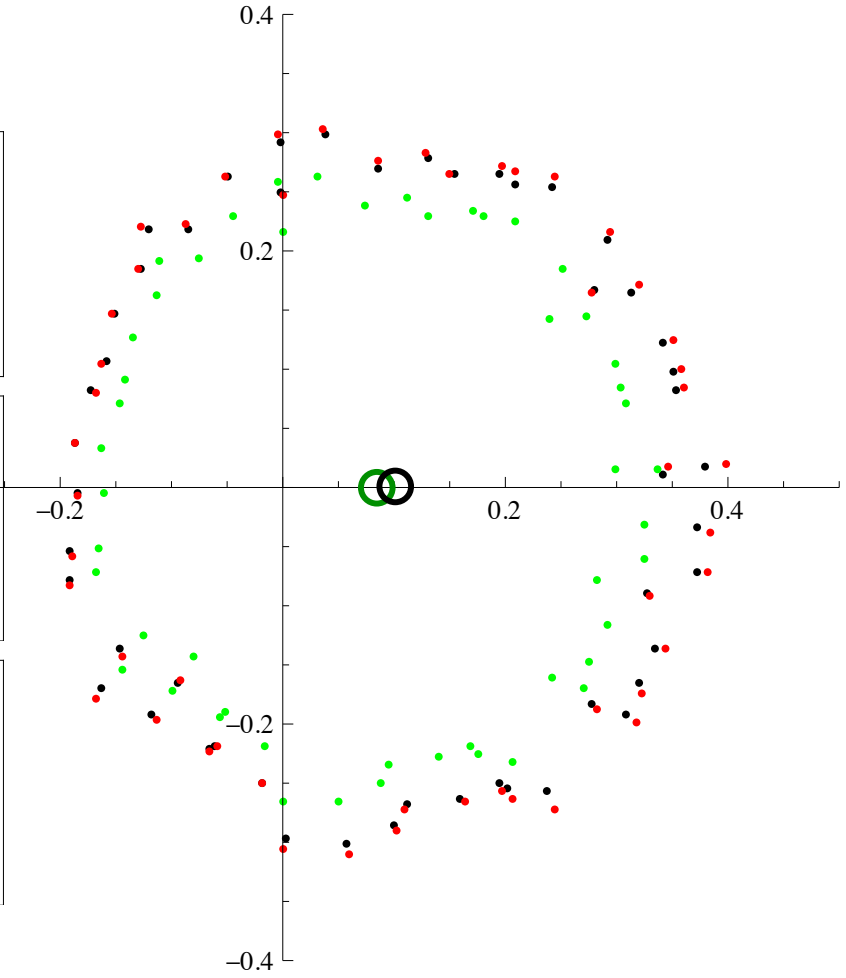


Black: No PSF applied

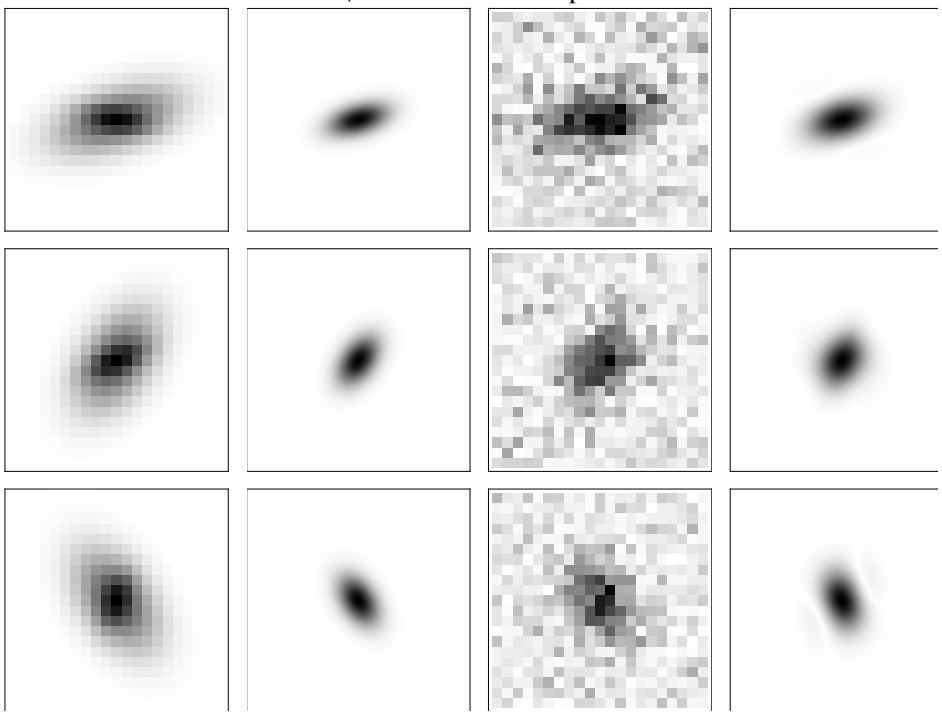
Red: PSF applied and corrected

Green: PSF applied but not corrected





Black: No PSF applied
Red: PSF applied and corrected
Green: PSF applied but not corrected


S/N = 30 PSF = 1 pixel

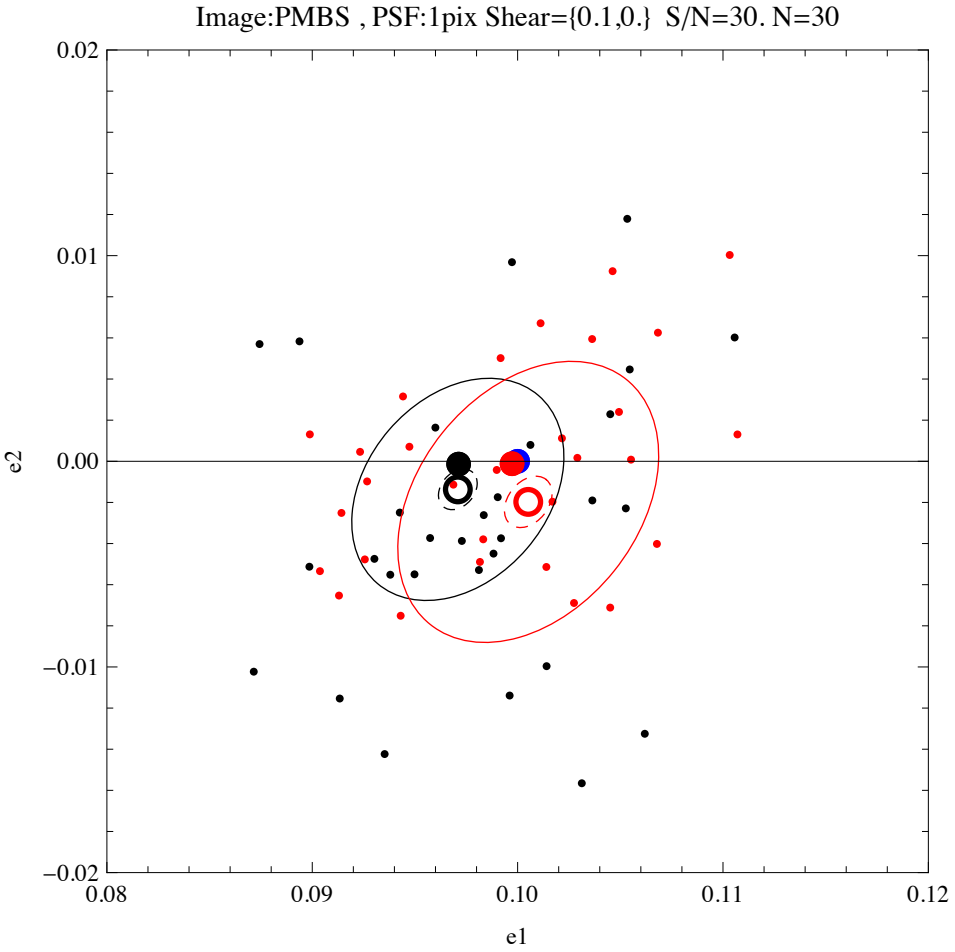
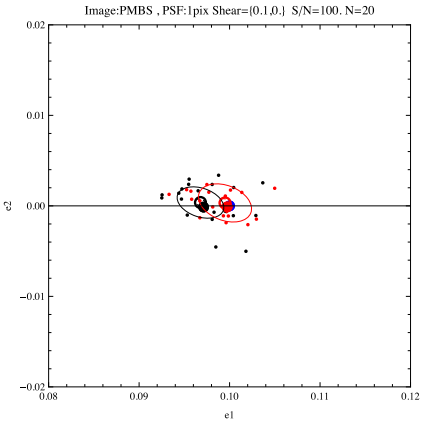
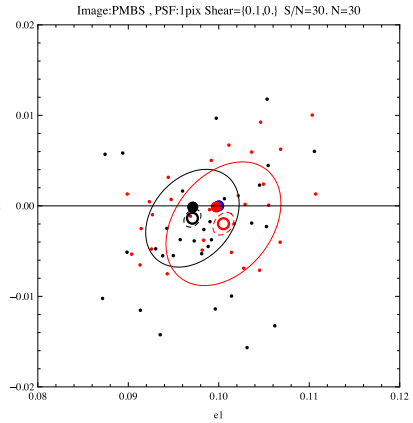


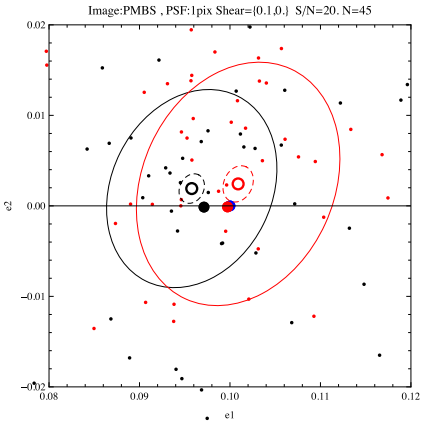
Image:PMBS , PSF:1pix Shear={0.1,0.} S/N=30. N=30
0.02
0.01
0.00
−0.01
−0.02
e2
0.08
0.09
0.10
0.11
0.12
e1

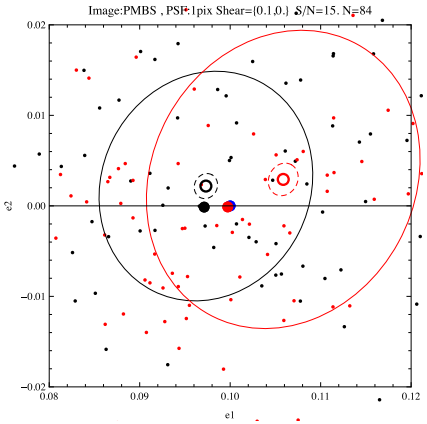
S/N = 100 30 20 15

PSF 1.0 Pixel

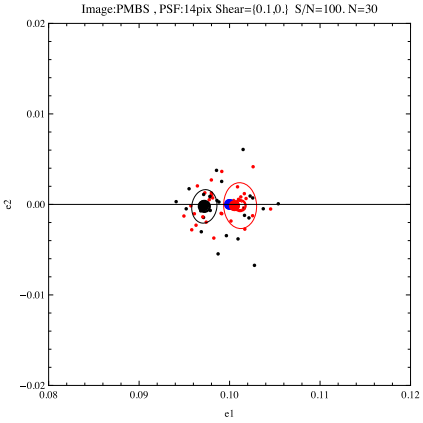


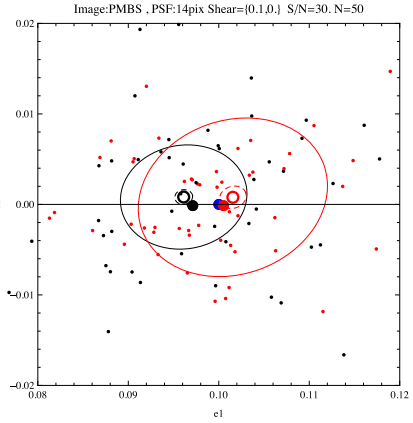


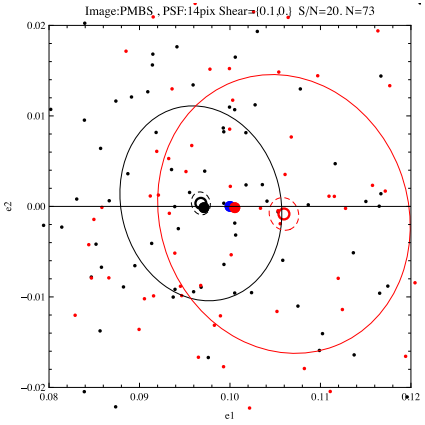


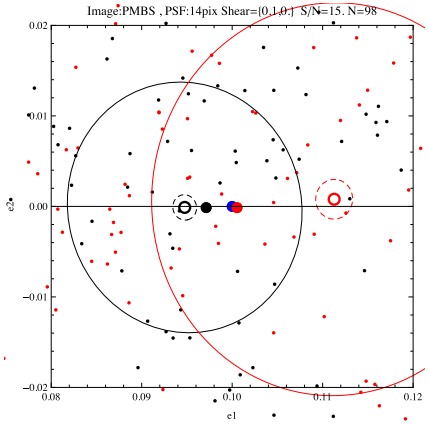


1.4 Pixel

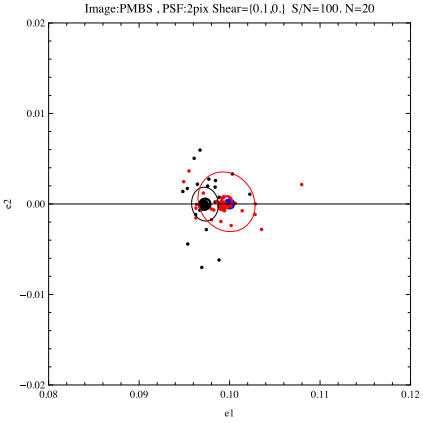


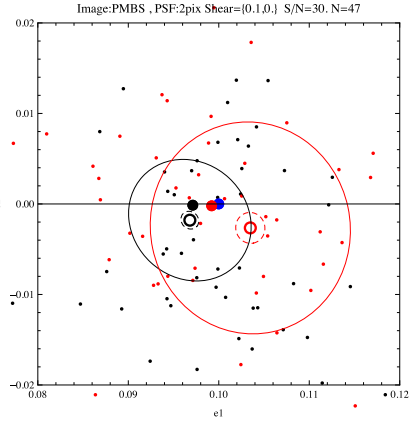


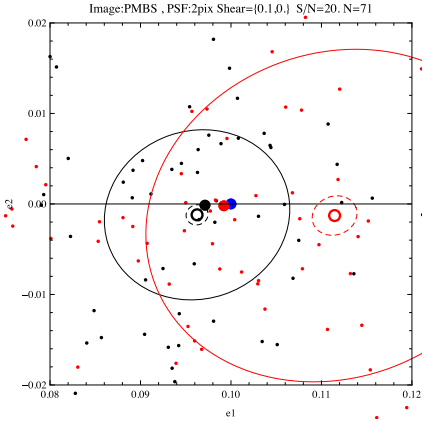


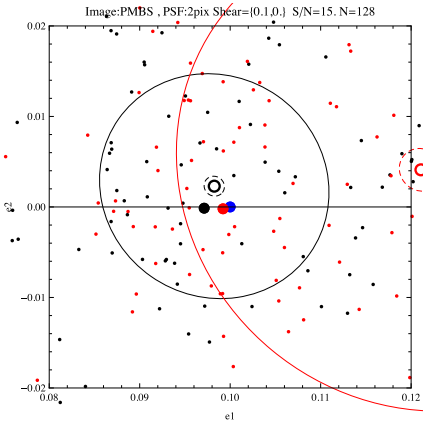


2.0 Pixel





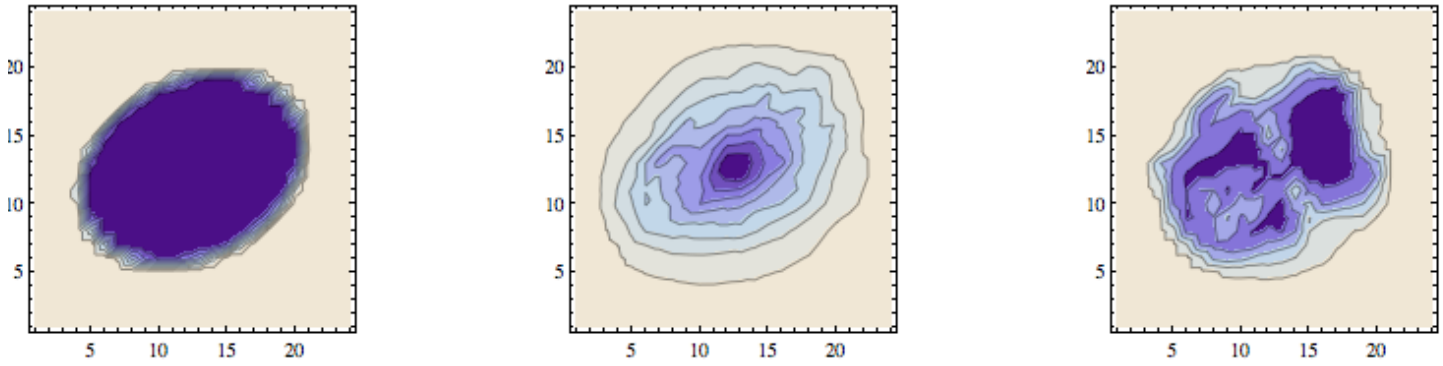




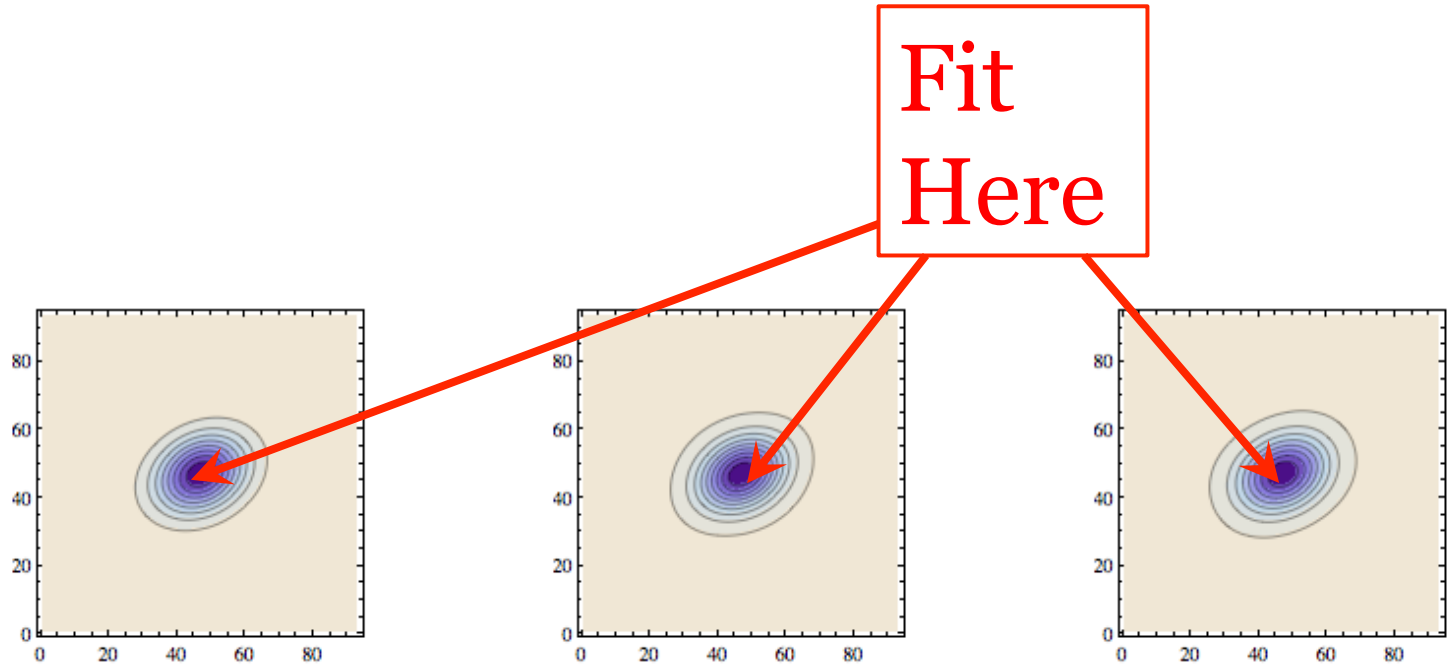
Summary

- In principle the autocorrelate has the same shear transform and PSF properties as an image
- In practice the autocorrelate results in a smooth profile, readily fit-able and preserving ellipticity even in the worst noise conditions
- Excellent noise resistance for shear estimator in absence of PSF
- With “quickie” faux-variance scheme, see very good PSF correction performance right out of the box before any optimizations or corrections

Central “cap” preserves ellipticity

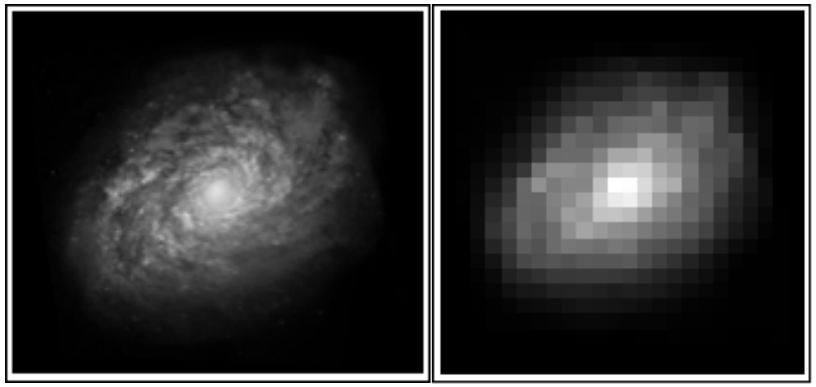


Original



Second auto - correlation

Fitting > Moments



How do we go from an image to measures of (e1,e2) estimators?

Two basic approaches:

Moments: e.g. covariance matrix

- No assumption of detailed galaxy shape
- Bias from clipping in tails
- Bad noise performance, or use weighting (unknown, tricky)

Fitting: with parameterized form

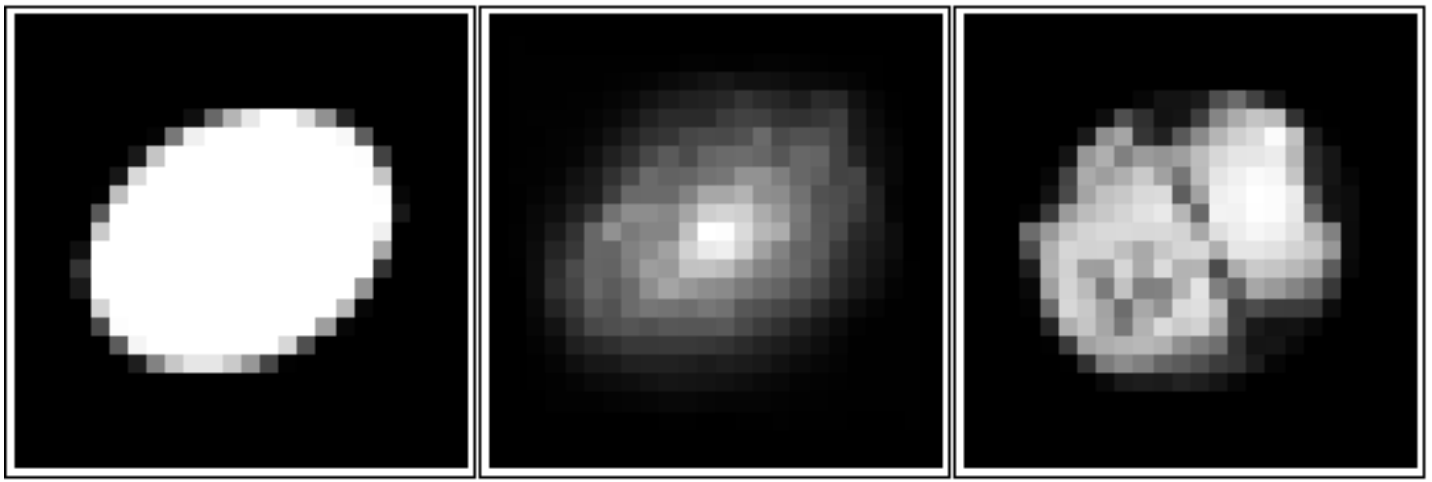
- Works on any piece of image
- Better noise resistance; but potential noise bias
- True shapes unknown; noise effect depends on shape (Great3)

Four experiments:

Using auto-correlation/convolution + fitting technique:

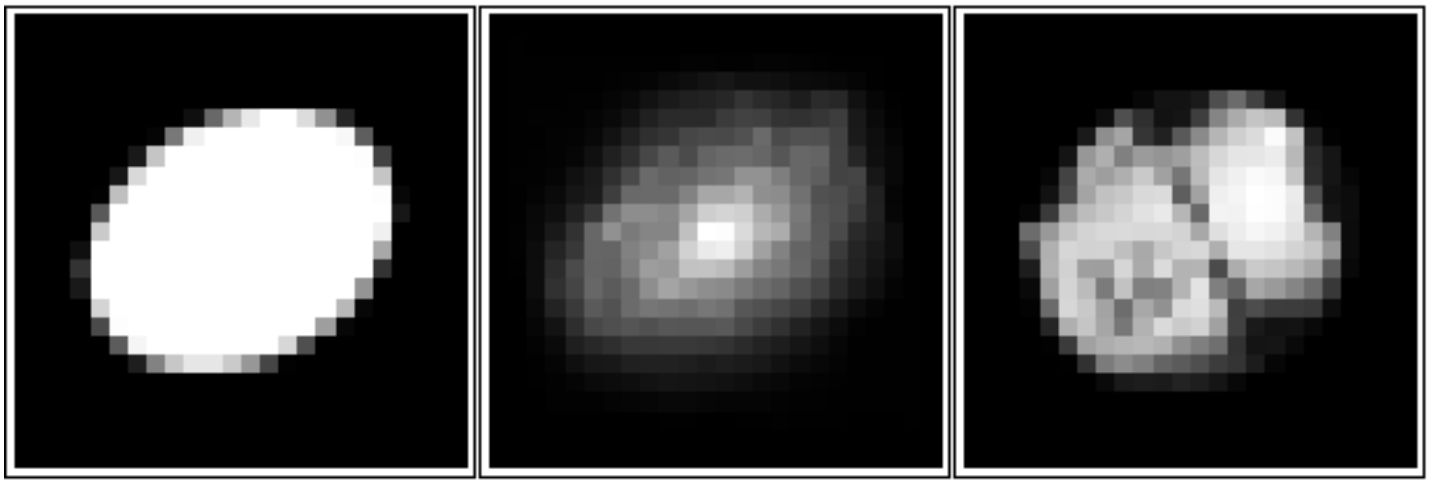
- Look at three very different images with same eccentricity, see how noise performance varies
- Look at noise behavior at high ellipticity
- Using Gaussian images, look at resistance to tail clipping
- Shear an assortment of Gaussians, look at shear recovery on average

E1: Meet our contestants



DISH NGC4414 RMN1970

E1: Meet our contestants



DISH NGC4414 RMN1970

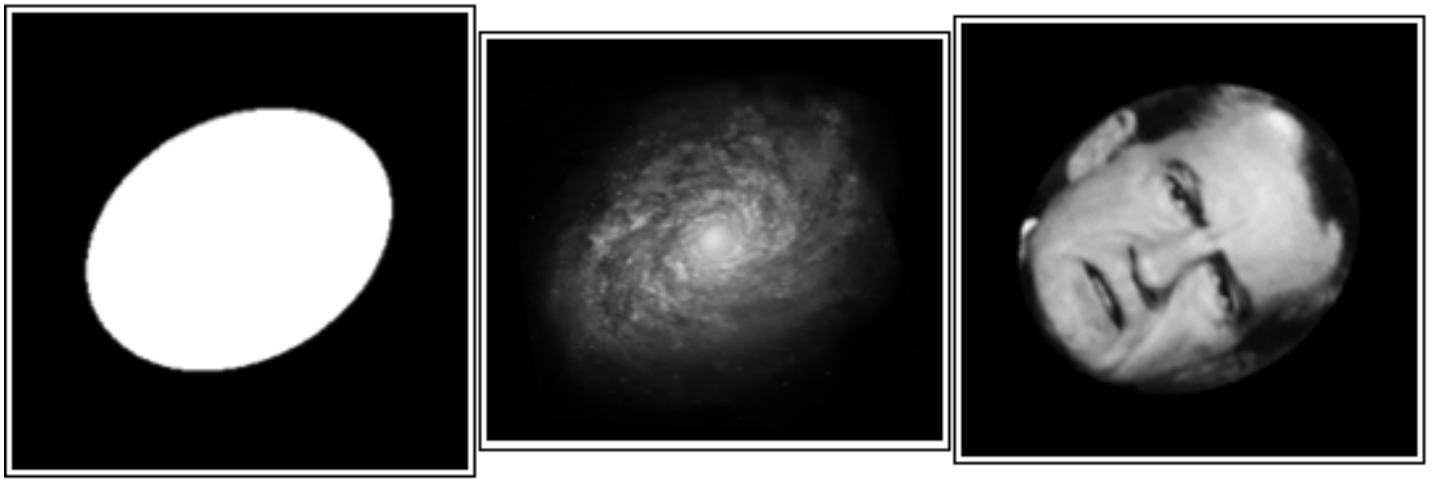
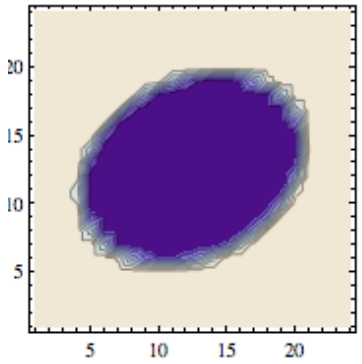
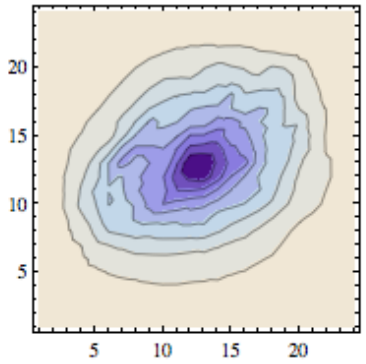
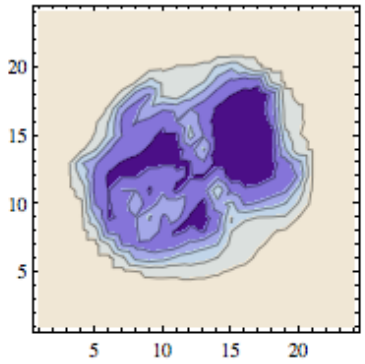


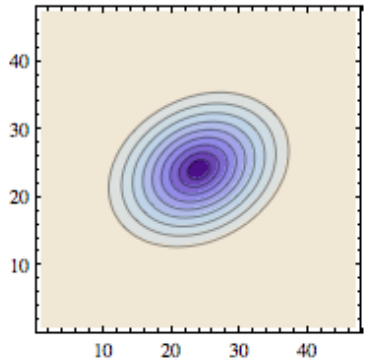
Image processing, second order

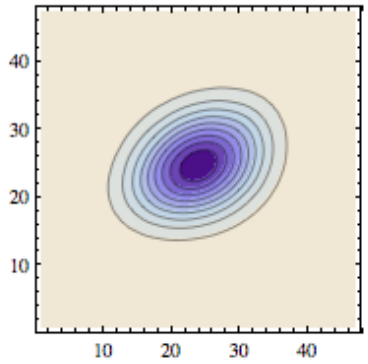


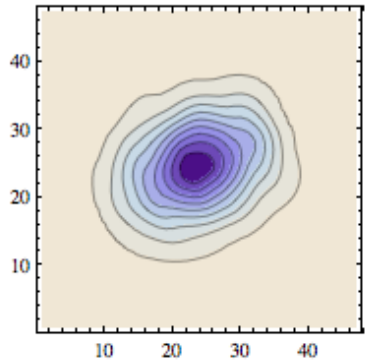




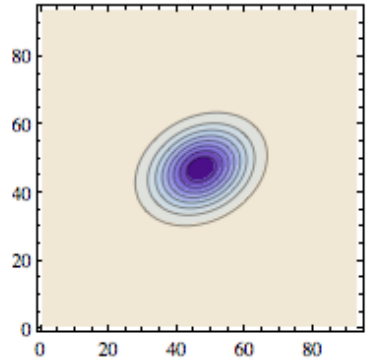
Original

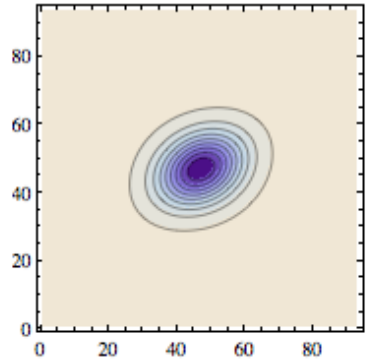


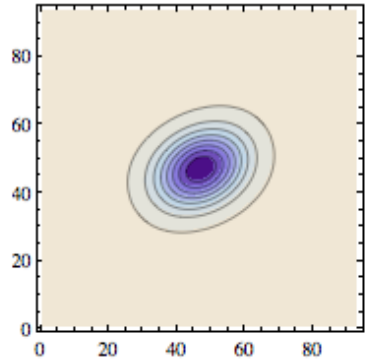




First auto - correlation

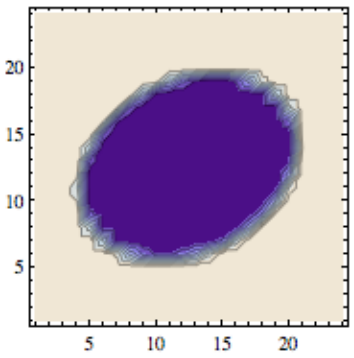


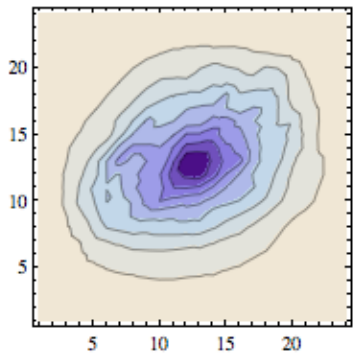


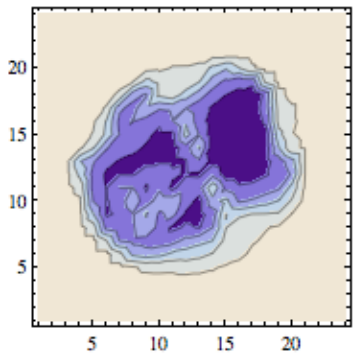


Second auto - correlation

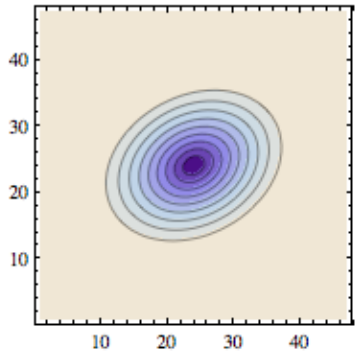
Image processing, first order

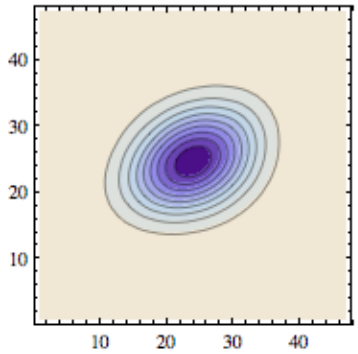


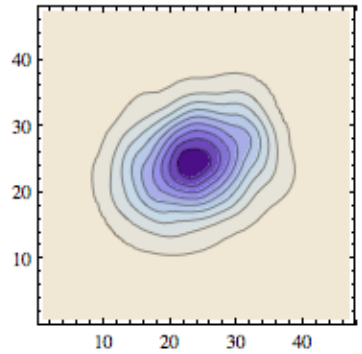




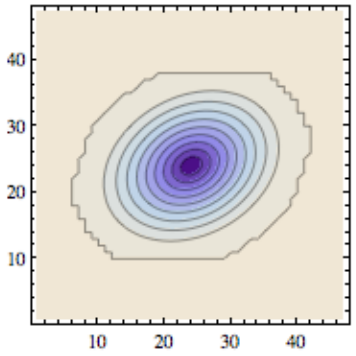
Original

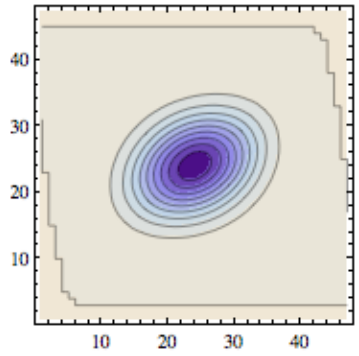


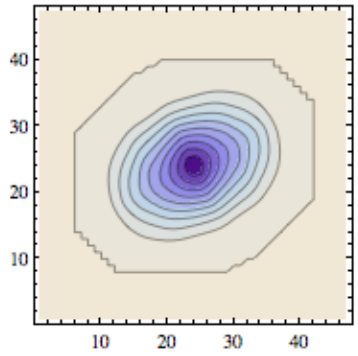




Auto - convolution







Auto - correlation